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SYM Description of SFT Hamiltonian in a PP-Wave Background

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Abstract

We compute string field theory Hamiltonian matrix elements and compare them with matrix elements of the dilatation operator in gauge theory. We get precise agreement between the string field theory and gauge theory computations once the correct cubic Hamiltonian matrix elements in string field theory and a particular basis of states in gauge theory are used. We proceed to compute the matrix elements of the dilatation operator to order g_2^2 in this same basis. This calculation makes a prediction for string field theory Hamiltonian matrix elements to order g_2^2, which have not yet been computed. However, our gauge theory results precisely match the results of the recent computation by Pearson et al. of the order g_2^2 Hamiltonian matrix elements of the string bit model.

1 Introduction

Recently, Berenstein, Maldacena and Nastase (BMN) [1] have discovered a particular limit of the AdS/CFT correspondence – known as the Penrose limit – in which string theory can be solved to all orders in α' [2, 3]. The limiting geometry, which is the plane wave in [4], has very different global properties compared to those of $AdS_5 \times S^5$. Since the Penrose limit focuses on the geometry in the vicinity of a particular massless particle in $AdS_5 \times S^5$, which is very massive when measured by an AdS_5 observer, information about the asymptotic AdS_5 time-like boundary is lost in the limit, and the conformal boundary of the plane wave degenerates to a null line [5]. This peculiar degeneration makes it challenging to find a precise holographic map between string theory and gauge theory. On the other hand, the limit performed on the geometry can also be realized on the dual gauge theory which leads BMN to conjecture that a certain sector of $\mathcal{N} = 4$ SYM captures the dynamics of string theory in the plane wave background.

A crucial step in making the correspondence precise is to identify the charges of string states in space-time with the charges carried by gauge theory operators. The identification is given by

$$\frac{1}{\mu}H = (\Delta - J), \qquad\qquad \mu R^2 P^+ = J, \tag{1.1}$$

where H is the generator of x^+ translations, P^+ is the generator of x^- translations, Δ is the generator of dilatations and J is a $U(1)_R$ charge. In the limit, the gauge theory truncates to the BMN sector of operators carrying large $U(1)_R$ charge $J \sim \sqrt{N}$ in the $N \to \infty$ limit. In this double scaling limit we keep the following parameters fixed

$$\lambda' = \frac{g^2 N}{J^2} = \frac{1}{(\mu p^+ \alpha')^2}, \qquad g_2 = \frac{J^2}{N} = 4\pi g_s (\mu p^+ \alpha')^2, \tag{1.2}$$

which effectively count quantum loops [1, 6] and non-planar corrections [7, 5, 8], respectively.

Even in the absence of string interactions, the BMN proposal makes a remarkable prediction about the spectrum of anomalous dimensions of BMN operators in the gauge theory in terms of the free string spectrum on the plane wave. This prediction was analyzed perturbatively in the regime where $\lambda' \ll 1$ in [1, 6] and confirmed to all orders using superconformal invariance in [9].

In the light-cone gauge, in which the plane wave string theory can be readily analyzed, the free Hamiltonian receives string corrections

$$H = H_2 + g_2 H_3 + \cdots \ , \tag{1.3}$$

which to lowest order result in non-trivial transition amplitudes between one and two-string states. In [8] a concrete proposal was made which relates these string theory transition amplitudes with three point functions of BMN operators in the gauge theory. Their proposal is therefore the first attempt to construct a holographic map at the interacting level between string theory in a plane wave and gauge theory.

This interesting proposal[1] was put to an explicit test by Spradlin and Volovich in [22] where some Hamiltonian matrix elements were computed using the string field theory vertex constructed previously in [23]. Exact agreement with the proposal given in [8] was reported. Unfortunately both the field theory and string theory computations suffer from errors which when taken into account invalidate the proposal. On the field theory side, operator mixing is more important than initially contemplated in [7, 8]. Three-point functions of single trace operators with order λ' interactions are not conformally invariant, but the correct form is restored after operator mixing is incorporated, as shown in [24, 25]. On the string field theory side, the prefactor acting on the delta functional overlap of three strings that enters the Hamiltonian H_3 [23] has a minus sign error, which was first reported by Pankiewicz [26]. Once matrix elements are recomputed with the corrected Hamiltonian H_3, which we calculate in section 2, agreement with field theory is lost. Therefore, what is the correct holographic map between string theory in the plane wave [4] and $\mathcal{N} = 4$ SYM at the interacting level?

The most straightforward way to proceed, which was first advocated in a paper by Gross, Mikhailov and Roiban [27], is to take the identification (1.1) between the string field theory Hamiltonian H and the generator of scale transformations $\Delta - J$ in $\mathcal{N} = 4$ SYM as the holographic map for all g_2. This holographic map therefore identifies Hamiltonian matrix elements in string field theory with those of the dilatation operator in gauge theory. In order to test this identification one must find an explicit map between the Hilbert space of string states and the Hilbert space of states in the gauge theory. Moreover, in order for the comparison to be meaningful one must compute the matrix elements of these operators in a basis in which the Hilbert space inner product in gauge theory is the same as that in string field theory. The obvious inner product in the Hilbert space of string theory is the familiar

[1] Various aspects of this proposal were considered in [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

inner product where, for example, the one-string states are orthogonal to two-string states.

In gauge theory, the Hilbert space inner product is induced by the matrix of two-point functions of BMN operators[2]

$$|x|^{2\Delta_0}\langle O_A \bar{O}_B\rangle = G_{AB} + \Gamma_{AB}\ln(x^2\Lambda^2)^{-1}, \tag{1.4}$$

where G_{AB} is the Hilbert space inner product and Γ_{AB} is the matrix of anomalous dimensions. Unlike with the usual Hilbert space inner product in string field theory, which remains diagonal to all orders in g_2, perturbative corrections in gauge theory induce operator mixing at each order in g_2 in perturbation theory and the Hilbert space inner product is no longer diagonal. Direct comparison with string field theory calculations requires correcting for operator mixing systematically, order by order in the g_2, expansion by making G_{AB} orthonormal via a change of basis.

In order to relate string theory to gauge theory calculations via (1.1) we must calculate the matrix elements of the dilatation operator. It is straightforward to show that the matrix elements of the dilatation operator between states created by BMN operators are given by the matrix of anomalous dimensions[3]

$$\langle O_A|(\Delta - J)|O_B\rangle = \Gamma_{AB}. \tag{1.5}$$

Comparison of matrix elements of H and Δ requires first making the gauge theory inner product orthonormal order by order in perturbation theory. We can accomplish this by finding a new basis of operators $\tilde{O}_A = U_{AB}O_B$ such that they are orthonormal

$$UGU^\dagger = 1. \tag{1.6}$$

When $g_2 = 0$ the correct identification between string states and gauge theory operators was given by BMN. Namely, an n-string state is described by an n-trace operator. Once g_2 corrections are taken into account this identification has to be modified. Therefore, the precise mapping between string field theory states $|s_A\rangle$ and gauge theory states $|\tilde{O}_A\rangle$ when $g_2 \neq 0$ is

[2]In this formula the various rows and columns describe single trace, double trace, etc components. A more complete characterization of this matrix is given in section 3.

[3]Since BMN operators are BPS or nearly BPS the contribution from the action of Δ coming from the bare dimension of the operator is cancelled by the contribution from the R-charge in (1.5).

given by[4]

$$|s_A\rangle \to |\tilde{O}_A\rangle = U_{AB}|O_B\rangle, \qquad \langle s_A|s_B\rangle = \langle \tilde{O}_A|\tilde{O}_B\rangle = \delta_{AB}. \tag{1.7}$$

In section 3 we give an expression for the change of basis to order g_2^2. Once the right basis is found, the holographic map reads

$$\frac{1}{\mu}\langle s_A|H|s_B\rangle = \langle \tilde{O}_A|(\Delta - J)|\tilde{O}_B\rangle = (U\Gamma U^\dagger)_{AB}. \tag{1.8}$$

The authors of [27], where this proposal was first made, concluded that this proposal is equivalent to the one given in [8] involving three point functions. However, we find that the two holographic maps are not the same and that the one given in (1.8) is the correct one.

An important subtlety in the holographic map (1.8) is that the orthonormalization procedure of the gauge theory inner product is not unique, namely the transformation matrix U that makes G_{AB} orthonormal is not unique[5]. We uniquely fix the form U to order g_2 by demanding that the dilatation operator matrix elements agree with the matrix elements of the corrected string field theory Hamiltonian that we calculate in section 2. We then evaluate U to order g_2^2 which via (1.1) makes a non-trivial prediction for string field theory Hamiltonian matrix elements to order g_2^2 which have not yet been evaluated. Remarkably, this purely gauge theory result we present is reproduced by the order g_2^2 string bit Hamiltonian [28, 29] matrix elements recently computed in [30].

The plan of the rest of the paper is as follows. In section 2 we revisit the string field theory Hamiltonian H_3 and point out that there is an incorrect relative minus sign in [22][6]. We recompute Hamiltonian matrix elements with the corrected sign, which we will use in section 3. In section 3 we fix the form of the change of basis in gauge theory to order g_2 by demanding that (1.8) holds when the string field theory matrix elements in section 2 are used. We also evaluate the dilatation operator matrix elements to order g_2^2 by making a particular choice of basis to order g_2^2 which allows us to make a calculable prediction by using (1.8) about the string field theory Hamiltonian matrix elements to that order. We note that the final answer

[4]In light-cone string field theory the canonical normalization of states is the usual delta function normalization $\langle s'_A|s'_B\rangle = p_A^+\delta(p_A^+ - p_B^+) = J_A\delta_{J_A,J_B}$, so that $|s'_A\rangle = \sqrt{J_A}|s_A\rangle$. Therefore, when comparing string field theory results with gauge theory results we will have to take into account this normalization factor, since gauge theory states have unit norm [5, 8].

[5]This ambiguity was first pointed out in [27].

[6]This sign has been corrected in a recent revision.

agrees with the recent result in [30] found using the string bit formalism [28, 29]. We conclude in section 4.

Note Added

While this work was being finished, the paper [30] appeared on the archive. They also reexamine the duality proposal made in [8] when the corrected string field theory Hamiltonian is used and also conclude that once this is taken into account that the proposal in [8] no longer holds. Moreover, they also adopt the philosophy in [27] for comparing gauge theory and string theory and also get agreement.

2 SFT computation revisited

Before going into our gauge theory computation, let us perform the correct string field theory calculation that we will compare it to. This also corrects some errors in the previous literature. In string field theory, the Hilbert space is a direct product of ℓ-string states,

$$\mathcal{H} = \oplus_{\ell} \mathcal{H}_{\ell}. \tag{2.9}$$

All these states are orthogonal with respect to each other and they have an orthonormal inner product. The full Hamiltonian H, representing infinitesimal evolution along x^+, includes the freely propagating part, H_2, and an interaction part, H_3 and so on,

$$H = H_2 + g_2 H_3 + \cdots \ . \tag{2.10}$$

In the plane wave background the freely propagating part H_2 is simply the energy of an infinite collection of harmonic oscillators α^i_n and the three-string interaction part H_3 contains the delta functional overlap of three strings with a prefactor acting on it which is necessary to appropriately realize the supersymmetry algebra. The string states, which are dual to a certain class of two impurity BMN operators when $g_2 = 0$, are given by

$$|n\rangle = \alpha_n^{1\dagger}\alpha_{-n}^{2\dagger}|0,1\rangle \tag{2.11}$$

$$|n,y\rangle\rangle = \alpha_n^{1\dagger}\alpha_{-n}^{2\dagger}|0,y\rangle \otimes |0,1-y\rangle \tag{2.12}$$

$$|y\rangle\rangle = \alpha_0^{1\dagger}|0,y\rangle \otimes \alpha_0^{2\dagger}|0,1-y\rangle, \tag{2.13}$$

where the first state represents a single string and the other two represent two-string states. Here $|0, y\rangle$ is the one-string vacuum state carrying a fraction $0 < y < 1$ of the total longitudinal momentum p^+ of the multi-string state. The Hamiltonian matrix elements of these states are given by

$$\langle n|H_3|m, y\rangle\rangle \sim \big(F^+_{(1)|m|}F^+_{(3)|n|} + F^-_{(1)|m|}F^-_{(3)|n|}\big)\big(\bar{N}^{(13)}_{|m|,|n|} - \bar{N}^{(13)}_{-|m|,-|n|}\big), \quad \text{if } mn > 0 \tag{2.14}$$

$$\langle n|H_3|m, y\rangle\rangle \sim \big(F^+_{(1)|m|}F^+_{(3)|n|} - F^-_{(1)|m|}F^-_{(3)|n|}\big)\big(\bar{N}^{(13)}_{|m|,|n|} + \bar{N}^{(13)}_{-|m|,-|n|}\big), \quad \text{if } mn < 0 \tag{2.15}$$

$$\langle n|H_3|y\rangle\rangle \sim F^+_{(3)|n|}\big(F^+_{(1)0}\bar{N}^{(23)}_{0,|n|} + F^+_{(2)0}\bar{N}^{(13)}_{0,|n|}\big), \quad \text{for } \forall n \neq 0 \tag{2.16}$$

where $F_{m(r)}$ comes from the prefactor and the Neumann matrices $\bar{N}^{(rs)}_{mn}$ come from the delta functional overlap. The negative modes are related to the positive modes by[7] (here $m, n > 0$)

$$\bar{N}^{(rs)}_{-m,-n} = -\big(U_{(r)}\bar{N}^{(rs)}U_{(s)}\big)_{m,n}, \tag{2.17}$$

$$F^-_{(r)m} = i\big(U_{(r)}F^+_{(r)}\big)_m, \tag{2.18}$$

with $\big(U_{(r)}\big)_{m,n} = \delta_{m,n}\Big(\sqrt{m^2 + (\mu p^+_{(r)}\alpha')^2} - \mu p^+_{(r)}\alpha'\Big)/m$. Note that in (2.18) we have an extra factor of i compared to the original literature [22][8], which was first pointed out in [26]. Using (2.17) and (2.18), we can show that both (2.14) and (2.15) reduce to the same expression.

In order to compare these answers with perturbative gauge theory we must analyze the $\mu \to \infty$ limit of (2.14), (2.15) and (2.16). The $\mu \to \infty$ behavior of both the prefactor and the Neumann matrices was evaluated in [22]. In the $\mu \to \infty$ limit, (2.14), (2.15) and (2.16) yield[9]

$$\frac{1}{\mu}\langle n|H_3|m, y\rangle\rangle = \frac{\lambda'}{2\pi^2}(1-y)\sin^2(n\pi y), \tag{2.19}$$

$$\frac{1}{\mu}\langle n|H_3|y\rangle\rangle = -\frac{\lambda'}{2\pi^2}\sqrt{y(1-y)}\sin^2(n\pi y). \tag{2.20}$$

These corrected results invalidate the agreement previously found in the literature[10]. In the next section we will use these results to test the holographic proposal in (1.8) and will show that agreement is found for a particular choice of basis.

[7]We would like to thank A. Pankiewicz for informing us of the correct factor of i in the second formula.

[8]More explicitly, formulas (3.15) and (3.21) in the original version of [22] should have an extra factor of i. This invalidates the evaluation of (4.4) [22].

[9]The precise overall numerical factor of the cubic string field theory Hamiltonian is not known. It is fixed by comparing with the gauge theory calculation in the next section.

[10]Also due to this extra i, the result proven in [19] that the prefactor reduces to energy difference between incoming and outgoing states should be modified. Instead it reduces to the energy difference in cos modes minus that in sin modes that appear in the worldsheet Fourier decomposition.

3 Orthonormalization and comparison with SFT

In this section, we make a change of operator basis such that the gauge theory inner product is orthonormal order by order in g_2. We then compute the matrix elements of the operator $\Delta - J$ in this basis. An important subtlety in this procedure is that the basis change that makes the Hilbert space inner product G orthonormal is not unique. The leading term at $g_2 = 0$ is uniquely fixed by the original correspondence explained in [1] between single/double string states and single/double trace operators. When $g_2 \neq 0$ the field theory operators start mixing and the matrix with which we make the inner product orthonormal is not unique, due to the familiar ambiguity when diagonalizing a matrix. We propose to fix this ambiguity in the change of basis by taking seriously the proposal in (1.8) and demanding that exact agreement with the corrected string field theory results computed in the previous section is obtained. Exact agreement is obtained for a unique choice of basis. This particular choice is the unique choice for which the transformation matrix is symmetric and real to order g_2. We then proceed to calculate to next to leading order, to order g_2^2. Here there is also an ambiguity in the orthonormalization procedure. If we make the strong assumption that the transformation matrix is still symmetric and real to this order we can uniquely determine the transformation matrix to order[11] g_2^2. Given these assumptions we can calculate explicitly the order g_2^2 matrix element between the orthonormal basis states which reduce when $g_2 = 0$ to single trace operators. This result gives a prediction using the proposal (1.8) for light-cone string field theory matrix elements to order g_2^2 involving single strings, which has not been constructed. Despite this incompleteness of light-cone string field theory, the matrix elements we predict from the gauge theory computation exactly match the order g_2^2 matrix elements computed recently in [30] using the string bit formalism [28, 29]. It would be very desirable to understand more precisely the relation between light-cone string field theory and the string bit formalism.

The particular set of gauge theory operators that we are interested in are the following single trace and double trace operators[12]

$$\mathcal{O}^J = \frac{1}{\sqrt{JN^J}} \mathrm{Tr} Z^J, \tag{3.21}$$

$$\mathcal{O}^J_{(i)} = \frac{1}{\sqrt{N^{J+1}}} \mathrm{Tr}\left(\phi_i Z^J\right), \tag{3.22}$$

[11] This choice is compatible with the proposal in [30].

[12] Here we are using the notation in [24].

$$\mathcal{O}_n^J = \frac{1}{\sqrt{JN^{J+2}}} \sum_{l=0}^{J} e^{2\pi i l n/J} \mathrm{Tr}\left(\phi_1 Z^l \phi_2 Z^{J-l}\right), \tag{3.23}$$

$$\mathcal{T}_p^{J,y} = :\mathcal{O}_p^{y\cdot J} \mathcal{O}^{(1-y)\cdot J}:, \tag{3.24}$$

$$\mathcal{T}^{J,y} = :\mathcal{O}_{(1)}^{y\cdot J} \mathcal{O}_{(2)}^{(1-y)\cdot J}:. \tag{3.25}$$

We need to make the inner product G appearing in the matrix of two-point functions in (1.4) orthonormal and eventually compute the matrix elements of Γ in the orthonormal basis (1.8). Both matrices G_{AB} and Γ_{AB} have a systematic expansion in powers of g_2

$$G = \mathbf{1} + g_2 G^{(1)} + g_2^2 G^{(2)} + \mathcal{O}(g_2^3), \tag{3.26}$$

$$\Gamma = \Gamma^{(0)} + g_2 \Gamma^{(1)} + g_2^2 \Gamma^{(2)} + \mathcal{O}(g_2^3). \tag{3.27}$$

In the following we split these matrices into 3×3 blocks representing matrix elements involving $\mathcal{O}_n^J$, $\mathcal{T}_p^{J,y}$ and $\mathcal{T}^{J,y}$ respectively. The indices $(n, m, \cdots)$ denote the worldsheet momentum of the single trace BMN operators like, for example, $\mathcal{O}_n^J$. The double indices $(py, qz, \cdots)$ represent for example the worldsheet momentum and light-cone momentum fraction of the double trace operators $\mathcal{T}_p^{J,y}$, while $(y, z, \cdots)$ represent the fraction of momentum carried by the operator $\mathcal{T}^{J,y}$. These matrices have been computed in [24, 25]. They are given by[13]:

$$G = \mathbf{1} + g_2 \begin{pmatrix} 0 & C_{n,qz} & C_{n,z} \\ C_{py,m} & 0 & 0 \\ C_{y,m} & 0 & 0 \end{pmatrix} + g_2^2 \begin{pmatrix} M^1_{n,m} & 0 & 0 \\ 0 & \langle ? \rangle & \langle ? \rangle \\ 0 & \langle ? \rangle & \langle ? \rangle \end{pmatrix}, \tag{3.28}$$

$$\frac{\Gamma}{\lambda'} = \begin{pmatrix} n^2 \delta_{n,m} & 0 & 0 \\ 0 & \frac{p^2}{y^2}\delta_{p,q}\delta_{y,z} & 0 \\ 0 & 0 & 0 \end{pmatrix} + g_2 \begin{pmatrix} 0 & \Gamma^{(1)}_{n,qz} & \Gamma^{(1)}_{n,z} \\ \Gamma^{(1)}_{py,m} & 0 & 0 \\ \Gamma^{(1)}_{y,m} & 0 & 0 \end{pmatrix} + g_2^2 \begin{pmatrix} nmM^1_{n,m} + \frac{1}{8\pi^2}\mathcal{D}^1_{n,m} & 0 & 0 \\ 0 & \langle ? \rangle & \langle ? \rangle \\ 0 & \langle ? \rangle & \langle ? \rangle \end{pmatrix}. \tag{3.29}$$

The explicit form of the matrix elements are summarized in the Appendix and we denote by $\langle ? \rangle$ matrix elements that have not yet been computed. Luckily, we will not need them for our computations. Finding them is, however, an important enterprise since via the holographic map (1.8) they predict yet unknown matrix elements in string field theory, like the order g_2^2 Hamiltonian matrix element of a two-string state[14].

[13] We would like to thank U. Gursoy for reminding us that the off-diagonal elements of the matrix of two-point functions involving double-trace operators are non-zero at order g_2^2.

[14] One should also compute, however, the mixing between double and triple trace operators to get this result.

Let us now apply a linear transformation U to make the inner product orthonormal. We require that

$$UGU^{\dagger} = \mathbf{1} \tag{3.30}$$

and solve this equation order by order. We can expand U in a power series in g_2 and express it as

$$U = \mathbf{1} + g_2 U^{(1)} + g_2^2 U^{(2)} + \mathcal{O}(g_2^3). \tag{3.31}$$

As explained in the beginning of this section we restrict our attention to symmetric and real matrices. This is motivated in part by the fact that the symmetric and real choice uniquely leads to exact agreement with string field theory via (1.8) to order g_2 as we will see below. Clearly, having a better understanding of why this choice works is very desirable. Therefore, by assuming that U is a symmetric and real matrix, we need to solve (3.30). Solving this equation order by order we get

$$U^{(1)} = -\frac{1}{2}G^{(1)}, \tag{3.32}$$
$$U^{(2)} = -\frac{1}{2}G^{(2)} + \frac{3}{8}\left(G^{(1)}\right)^2. \tag{3.33}$$

In the new orthonormal basis, Γ is transformed to

$$\tilde{\Gamma} = U\Gamma U^{\dagger}. \tag{3.34}$$

We can determine Γ order by order in g_2 by expanding

$$\tilde{\Gamma} = \tilde{\Gamma}^{(0)} + g_2\tilde{\Gamma}^{(1)} + g_2^2\tilde{\Gamma}^{(2)} + \mathcal{O}(g_2^3). \tag{3.35}$$

The matrix of anomalous dimensions in the new basis is therefore

$$\tilde{\Gamma}^{(0)} = \Gamma^{(0)}, \tag{3.36}$$
$$\tilde{\Gamma}^{(1)} = \Gamma^{(1)} - \frac{1}{2}\{G^{(1)}, \Gamma^{(0)}\}, \tag{3.37}$$
$$\tilde{\Gamma}^{(2)} = \Gamma^{(2)} - \frac{1}{2}\{G^{(2)}, \Gamma^{(0)}\} - \frac{1}{2}\{G^{(1)}, \Gamma^{(1)}\} + \frac{3}{8}\{\left(G^{(1)}\right)^2, \Gamma^{(0)}\} + \frac{1}{4}G^{(1)}\Gamma^{(0)}G^{(1)}. \tag{3.38}$$

Using (3.28) and (3.29) we can evaluate (3.37) to be

$$\tilde{\Gamma}^{(1)} = \begin{pmatrix} 0 & \tilde{\Gamma}^{(1)}_{n,qz} & \tilde{\Gamma}^{(1)}_{n,z} \\ \tilde{\Gamma}^{(1)}_{py,m} & 0 & 0 \\ \tilde{\Gamma}^{(1)}_{y,m} & 0 & 0 \end{pmatrix}, \tag{3.39}$$

where

$$\tilde{\Gamma}^{(1)}_{n,py} = \tilde{\Gamma}^{(1)}_{py,n} = \lambda' \frac{\sqrt{1-y}}{\sqrt{Jy}} \frac{\sin^2(\pi n y)}{2\pi^2}, \tag{3.40}$$

$$\tilde{\Gamma}^{(1)}_{n,y} = \tilde{\Gamma}^{(1)}_{y,n} = -\lambda' \frac{1}{\sqrt{J}} \frac{\sin^2(\pi n y)}{2\pi^2}. \tag{3.41}$$

We note that after using the proposed holographic map (1.8) that the gauge theory results (3.40), (3.41) match with the string field theory results (2.19), (2.20)[15].

We can now make a prediction about the order g_2^2 matrix elements in string field theory by using (1.8). In order to do that we must calculate the matrix of anomalous dimensions in the new basis to order g_2^2. By using (3.28), (3.29) we can perform the sums in (3.38) to get[16]

$$\tilde{\Gamma}^{(2)} = \begin{pmatrix} \tilde{\Gamma}^{(2)}_{n,m} & 0 & 0 \\ 0 & \langle ? \rangle & \langle ? \rangle \\ 0 & \langle ? \rangle & \langle ? \rangle \end{pmatrix}, \tag{3.42}$$

where[17]

$$\tilde{\Gamma}^{(2)}_{n,m} = \begin{cases} \dfrac{\lambda'}{32\pi^4} \left(\dfrac{3}{nm} + \dfrac{1}{(n-m)^2} \right) & \text{if } n \neq m, -m \\ \dfrac{\lambda'}{16\pi^2} \left(\dfrac{1}{3} + \dfrac{5}{2\pi^2 n^2} \right) & \text{if } n = m \\ -\dfrac{15\lambda'}{128\pi^4 n^2} & \text{if } n = -m \end{cases} \tag{3.43}$$

and $\langle ? \rangle$ are quantities that we cannot determine since the full matrix of two-point functions has not been computed to order g_2^2. We can nevertheless make the following prediction

$$\frac{1}{\mu} \langle n | H | m \rangle \Big|_{g_2^2} = \tilde{\Gamma}^{(2)}_{n,m}. \tag{3.44}$$

We should note that this quantity has not been yet been computed in light-cone string field theory. However, (3.44) exactly agrees with the recent proposal in [30] for the Hamiltonian matrix elements of the string bit Hamiltonian.

[15] As explained in a footnote in page 4, in order compare the string field theory answer with the gauge theory answer, one must divide the string result by $\sqrt{Jy(1-y)}$ so that both string field theory states and gauge theory states have unit norm.

[16] The formulas we need to compute the required sums are summarized in the Appendix.

[17] Numerically, this expression is the same as the the one in [8] for the non-nearest neighbor genus 1 single-trace two-point function.

4 Discussion

In this paper we have studied the gauge theory realization of string field theory Hamiltonian matrix elements. The answer, which was already anticipated in [27], is that these matrix elements correspond to matrix elements of the dilatation operator. Using the corrected string field theory results in section 2 we find a preferred basis of states which yields agreement between gauge theory and string theory calculations. Moreover, we make a prediction using a gauge theory computation for the Hamiltonian matrix elements of single string states to order g_2^2, which have not yet been computed. We, however, report precise agreement with the recent string bit [28, 29] Hamiltonian calculation presented in [30]. An outcome of the corrected string field theory calculation in section 2 is that the the proposal of [8] no longer holds. This paper gives evidence that the correct correspondence is between matrix elements of the string Hamiltonian and the dilatation operator in the gauge theory. This proposal suggests that the only observables that can be holographically computed in the plane wave string theory are gauge theory two-point functions [28, 27]. Nevertheless, operator mixing between multi-trace operators contains information about higher point functions in gauge theory. In string field theory, the Hamiltonian matrix elements compute the matrix of anomalous dimensions in the orthonormal basis via (1.8), which can be read from the gauge theory two-point functions.

The mapping between the string field theory and gauge theory Hilbert spaces is non-unique. By comparing the calculation of the dilatation operator matrix elements with the corrected string field theory Hamiltonian matrix elements in section 2 we fixed the ambiguity, which picks a particular basis of states in the gauge theory to be identified with string states. However, it would be very desirable to have a first principles explanation of why this choice is the correct one. In the recent paper [30] the authors motivate this choice by proving that the string bit Hamiltonian simplifies in this basis, that is, it truncates at finite order in g_2. It would be desirable to understand the uniqueness of the basis choice more directly. In this choice of basis the gauge theory computation we present in section 3 exactly agrees with the recent calculation [30] performed at order g_2^2 using the string bit [28, 29] Hamiltonian.

A fascinating open problem is to understand more precisely the relation between light-cone string field theory and the string bit formalism. The advantage of the string bit formalism is, as shown in [30], that the Hamiltonian truncates at order g_2^2. This however seems to raise a puzzle. The Hamiltonian matrix elements truncate at order g_2^2, which via the map (1.8) predicts that matrix elements of the dilatation operator in some basis truncate at order g_2^2 even though the Hilbert space inner product G_{AB} and the matrix of anomalous dimensions Γ_{AB} have corrections to all orders in g_2. It would be very interesting to study this prediction in detail. It still remains very desirable, however, to explicitly construct the light-cone string field theory Hamiltonian at order g_2^2 and explicitly verify that its matrix elements are those computed in (3.44).

Light cone string field theory and the string bit model have some complementary features. In the string bit formalism it is easier to compute the g_2^2 corrections, while the same problem is notoriously difficult in light-cone string field theory. On the other hand, in light-cone string field theory we can systematically evaluate $1/\mu$ corrections, which are hard to obtain in the string bit model. Computing these corrections is crucial in extending the duality beyond leading order in λ' in the gauge theory. In particular, the results in (2.14), (2.15) and (2.16) make non-trivial predictions about $\mathcal{O}(\lambda')$ corrections to the matrix of anomalous dimensions via (1.8). The factorization theorems proven in [31, 26] will be very useful in computing systematically all these corrections. It would also be very desirable to compute these corrections directly in gauge theory.

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Appendix

In this appendix, we present the definition of several matrices appearing in the main text and some useful formulae necessary for computing the matrix elements of Γ_{AB} in the new basis. They have been obtained from [24].

Matrix elements $\quad (|m| \neq |n|, m \neq 0, n \neq 0, p \in \mathbf{Z}, 0 < y < 1)$

- $$C_{n,py} = C_{py,n} = \frac{y^{3/2}\sqrt{1-y}}{\sqrt{J}\pi^2} \frac{\sin^2(\pi n y)}{(p-ny)^2} \tag{4.45}$$

  $$C_{n,y} = C_{y,n} = -\frac{1}{\sqrt{J}\pi^2} \frac{\sin^2(\pi n y)}{n^2} \tag{4.46}$$

- $$M^1_{n,n} = \frac{1}{60} - \frac{1}{24\pi^2 n^2} + \frac{7}{16\pi^4 n^4} \tag{4.47}$$

  $$M^1_{n,-n} = \frac{1}{48\pi^2 n^2} + \frac{35}{128\pi^4 n^4} \tag{4.48}$$

  $$M^1_{n,m} = \frac{1}{12\pi^2(n-m)^2} - \frac{1}{8\pi^4(n-m)^4} + \frac{1}{4\pi^4 n^2 m^2} + \frac{1}{8\pi^4 nm(n-m)^2} \tag{4.49}$$

- $$\Gamma^{(1)}_{n,py} = \Gamma^{(1)}_{py,n} = \left(\frac{p^2}{y^2} - \frac{pn}{y} + n^2\right) C_{n,py} \tag{4.50}$$

  $$\Gamma^{(1)}_{n,y} = \Gamma^{(1)}_{y,n} = n^2 C_{n,y} \tag{4.51}$$

- $$\mathcal{D}^1_{n,n} = \mathcal{D}^1_{n,-n} = \frac{2}{3} + \frac{5}{\pi^2 n^2} \tag{4.52}$$

  $$\mathcal{D}^1_{n,m} = \frac{2}{3} + \frac{2}{\pi^2 n^2} + \frac{2}{\pi^2 m^2} \tag{4.53}$$

Useful summation formulae

When one multiplies two matrices in (3.38), the following formulae are useful[18]:

- $$\sum_{p,y} C_{n,py} C_{py,m} = \frac{1}{J\pi^4} J \int_0^1 dy y^3 (1-y) \sin^2(\pi n y) \sin^2(\pi m y) \sum_{p=-\infty}^{\infty} \frac{1}{(p-nr)^2(p-mr)^2}$$
  $$= \begin{cases} \dfrac{1}{6\pi^2(n-m)^2} + \dfrac{1}{4\pi^4 n^2 m^2} + \dfrac{1}{\pi^4 nm(n-m)^2} - \dfrac{1}{4\pi^4(n-m)^4} & \text{if } n \neq m \\ \dfrac{1}{30} - \dfrac{1}{12\pi^2 n^2} + \dfrac{1}{2\pi^4 n^4} & \text{if } n = m \end{cases} \tag{4.54}$$

- $$\sum_{y} C_{n,y} C_{y,m} = \frac{1}{J\pi^4} J \int_0^1 dy \frac{\sin^2(\pi n y)}{n^2} \frac{\sin^2(\pi m y)}{m^2}$$
  $$= \begin{cases} \dfrac{1}{4\pi^4 n^2 m^2} & \text{if } n \neq m, -m \\ \dfrac{3}{8\pi^4 n^4} & \text{if } n = m, -m \end{cases} \tag{4.55}$$

- $$\sum_{p,y} \frac{p}{y} C_{n,py} C_{py,m} = \frac{1}{J\pi^4} J \int_0^1 dy y^2 (1-y) \sin^2(\pi n y) \sin^2(\pi m y) \sum_{p=-\infty}^{\infty} \frac{p}{(p-nr)^2(p-mr)^2}$$

[18] Similar identities can also be found in the Appendix of [25].

$$
= \begin{cases} (n+m)\left\{\frac{1}{12\pi^2(n-m)^2} + \frac{1}{4\pi^4 n^2 m^2} + \frac{1}{8\pi^4 nm(n-m)^2} - \frac{1}{8\pi^4(n-m)^4}\right\} & \text{if } n \neq m \\ \frac{n}{30} - \frac{1}{12\pi^2 n} + \frac{7}{8\pi^4 n^3} & \text{if } n = m \end{cases}
\tag{4.56}
$$

- $$\sum_{p,y} \frac{p^2}{y^2} C_{n,py} C_{py,m} = \frac{1}{J\pi^4} J \int_0^1 dy y(1-y) \sin^2(\pi n y) \sin^2(\pi m y) \sum_{p=-\infty}^{\infty} \frac{p^2}{(p-nr)^2(p-mr)^2}$$

$$
= \begin{cases} \frac{n^2+m^2}{12\pi^2(n-m)^2} + \frac{n^6+m^6-2nm(n^4+m^4)+n^3m^3}{4\pi^4 n^2 m^2 (n-m)^4} & \text{if } n \neq m \\ \frac{n^2}{30} + \frac{3}{2\pi^4 n^2} & \text{if } n = m \end{cases}
\tag{4.57}
$$

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